USE OF THE SEMIMOMENT METHOD TO SOLVE THE SHOCK LAYER RADIATIVE HEAT-TRANSFER PROBLEM

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A method is proposed for calculating heat transfer by radiation in a viscous shock layer, based on the of the semimoment method and the method of quasi-linearization.

An efficient method was proposed in [1] for solving the system of equations of motion of a selectively radiating gas in a shock layer, based on knowledge of the average absorption coefficients. However, the discontinuous nature of the average coefficient functions sometimes leads to instability in the calculation, which somewhat reduces the practical value of the method.

Below we describe an approach free from this defect. The method developed is based on the concept of eliminating calculations of selective radiative transfer from each cycle of the iteration process [1], and the use of the semimoment method [2] to solve the radiative transfer equation, and application of the quasilinearization method to calculate flow in the shock layer [3].

The choice of the semimoment method for integrating the transfer equation stems from its two under-mentioned properties which are important when one constructs a method for solving the total problem of flow of a radiating gas in a shock layer. The method is based on analytical integration of the radiative transfer equation with respect to the angular coordinate, yielding a system of ordinary differential equations for functions dependent only on the space coordinate. In addition, it differs favorably from other methods using preliminary analytical integration with respect to the angular coordinate (e.g., the method of moments) in that the semimoment functions of the radiative intensity are of constant sign and are finite.

The problem is formulated in full accord with [1], and therefore there is no need here to give the original system of equations describing flow of the gas in the high-temperature shock layer, nor to discuss the assumptions used.

According to the method of solution adopted, calculation of the spectral radiative intensity field J_λ from the equation

$$\mu J_{\lambda,y} = k_\lambda (J_{b,\lambda} - J_\lambda) \tag{1}$$

is replaced by defining the functions $M^{\pm}_{n,\lambda}$ (n = 0.1) satisfying the system of differential equations

$$\begin{aligned} M^{+}_{0,\lambda,y} &= 6\,k_\lambda (-M^{+}_{0,\lambda} + M^{+}_{1,\lambda} + \pi J_{b,\lambda}),\\ M^{+}_{1,\lambda,y} &= k_\lambda (-M^{+}_{0,\lambda} + 2\,\pi J_{b,\lambda}),\\ M^{-}_{0,\lambda,y} &= 6\,k_\lambda (M^{-}_{0,\lambda} + M^{-}_{1,\lambda} - \pi J_{b,\lambda}),\\ M^{-}_{1,\lambda,y} &= k_\lambda (-M^{-}_{0,\lambda} + 2\,\pi J_{b,\lambda}). \end{aligned} \tag{2}$$

The relation between the semimoment characteristics $M^{\pm}_{n,\lambda}$ and the spectral radiative intensity is the following:

$$M^{+}_{n,\lambda} = 2\,\pi \int_0^1 J_\lambda \mu^n d\mu,\quad M^{-}_{n,\lambda} = 2\,\pi \int_1^0 J_\lambda \mu^n d\mu. \tag{3}$$

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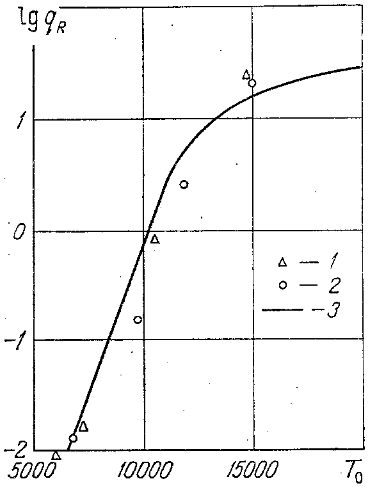

Fig. 1

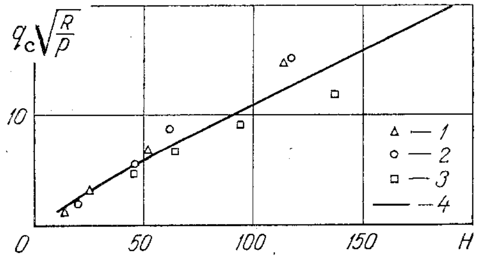

Fig. 2

Fig. 1. The integral radiative energy flux to the washed surface q_R for R = 0.1 m and p = 10^5 N/m^2 as a function of the stagnation temperature T_0: 1) this paper; 2) [1]; 3) [4]. The units of q_R are MW/m^2 and of T_0 °K.

Fig. 2. The convective heat flux to the washed surface q_c as a function of the stagnation enthalpy H: 1) this paper; 2) [1]; 3) [5]; 4) [4]. The units of $q_c\sqrt{Rp^{-1}}$ are MW/m$^{1.5}$·bar$^{0.5}$, and the units of H are MJ/kg.

The boundary conditions for the radiative intensity at the body surface and at the shock front are modified to account for Eq. (3).

In solving the problem of gas motion in a high-temperature shock layer it is very efficient to calculate the temperature and the radiative function fields, not in succession as was done in [1] by the discrete ordinate method, but in parallel, i.e., the equations of motion and energy conservation are solved simultaneously with the radiative transfer equation by the quasilinearization method [3]. This can be done if one divides the process of seeking a solution into two stages. In the first stage the system of equations (2) is solved in the selective formulation, and the effective integral absorption coefficients

$$\varkappa_n^{\pm} = \frac{\int\limits_0^\infty k_\lambda M_{n,\lambda}^{\pm}\, d\lambda}{M^{\pm}}; \tag{4}$$

$$\varkappa_p = \Big(\int\limits_0^\infty k_\lambda J_{b,\lambda} d\lambda\Big)\Big(\int\limits_0^\infty J_{b,\lambda} d\lambda\Big)^{-1} \tag{5}$$

are introduced, where $M_n^{\pm}$ are the integral moment characteristics, defined according to

$$M_n^{\pm} = \int\limits_0^\infty M_{n,\lambda}^{\pm}\, d\lambda. \tag{6}$$

We note that $\varkappa_p$ is the Planck mean absorption coefficient.

In the second stage we directly seek a solution of the system of gasdynamic equations and the system of moment equations, but the latter is already used in a certain "gray-gas" formulation obtained with the help of Eqs. (4) and (5):

$$M_{0,y}^{+} = 6\,(-\varkappa_0^{+} M_0^{+} + \varkappa_1^{+} M_1^{+} + \varkappa_p \sigma T^4),$$
$$M_{1,y}^{+} = -M_0^{+}\varkappa_0^{+} + 2\,\varkappa_p \sigma T^4, \tag{7}$$
$$M_{0,y}^{-} = 6\,(\varkappa_0^{-} M_0^{-} + \varkappa_1^{-} M_1^{-} - \varkappa_p \sigma T^4),\;\; M_{1,y}^{-} = -M_0^{-}\varkappa_0^{-} + 2\,\varkappa_p \sigma T^4.$$

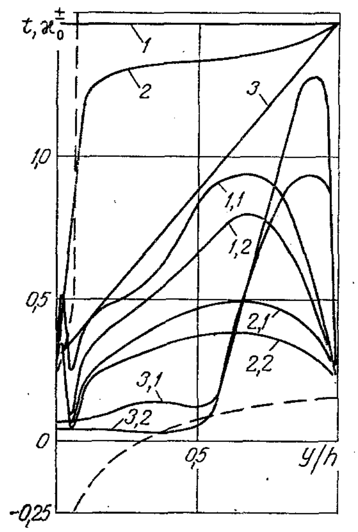

Fig. 3. Distribution of the temperature $T = t \cdot 10^4$ (curves 1, 2, and 3) and of the effective absorption coefficients in the compressed layer region: the first number corresponds to the temperature profile for which the effective integral coefficients were computed, while the second number is 1 for $\varkappa_0^+$ and 2 for $\varkappa_0^-$. The units of T are °K, and the units of $\varkappa_0^{\pm}$ are 1/cm.

One should note the following two circumstances which are evidence that the use here of the semimoment method is well founded:

1) It follows from the definition of the semimoment radiative characteristics given by Eq. (3) that the effective absorption coefficients introduced according to Eqs. (4) and (5) are constant in sign and finite, while the functions of the mean absorption coefficients determined in [1] by the method of discrete ordinates have points of discontinuity of the second kind;

2) To solve the selective part of the problem, one can use the solution of system (2) in integral form, which eliminates the question of possible direct integration of this system in the same frequency ranges where the optical thicknesses of the layer are large enough.

It should also be stressed that, as in [1], by using this approach to solve the problem one can avoid seeking a solution of the selective problem from a large number of iterations.

By way of an example of the use of this method, we computed the flow of radiating air in the shock layer near the stagnation line for conditions analogous to those in [1]. We used exactly the same thermodynamic properties of air, viscosity coefficients, Prandtl numbers, and spectral absorption coefficients of air. The only difference from [1] was that the air absorption coefficient was approximated by a piecewise-constant function in eight sections: λ = 0.02 – 0.1 – 0.11 – 0.125 – 0.22 – 0.335 – 0.57 – 1.35 – 4 μm.

Figures 1 and 2 show the results of calculations of the radiative and convective fluxes evaluated in this paper and also by other authors. Figure 3 shows profiles of the effective absorption coefficients and the corresponding temperature distribution through the shock layer. The broken lines in Fig. 3 show qualitatively the profiles of mean absorption coefficients introduced in [1].

One should note as an overall characteristic of this method of solving the problem that the program written for the computation on the BÉSM-6 computer (in Fortran language) showed adequately high operational qualities — fast convergence of the iterative process (3-7 iterations), and acceptable speed (~5 min).

NOTATION

I_λ, $J_{e,\lambda}$, spectral radiative intensity of the medium and of a perfect blackbody; $\mu = \cos\theta$; θ, angle between the direction of the y axis, normal to the surface, and the ambient direction of propagation of the radiation; k_λ, volume spectral absorption coefficient; $(\ )_y$, differentiation with respect to y; T, gas temperature; σ, Stefan–Boltzmann constant; λ, wavelength of the radiation; R, radius of blunting of the body; p, pressure in the shock layer; H, stagnation enthalpy; q_R, integral radiative heat flux to the surface; q_c, convective heat flux; h, shock layer thickness.

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SPATIAL NONSTATIONARY HEAT-CONDUCTION PROBLEM FOR A PRISM WITH A COORDINATE-DEPENDENT HEAT-TRANSFER COEFFICIENT

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We present an efficient method for the determination of three-dimensional non-steady-state fields of bodies of simple shapes, when the heat-transfer coefficient from their surface changes locally.

We consider an isotropic semiinfinite rectangular prism $0 \leqslant z \leqslant \infty$, $0 \leqslant x \leqslant x_0$, $0 \leqslant y \leq y_0$. Through the face z = 0 of the prism, convective heat exchange takes place with the inhomogeneous external medium. The temperature of the external medium, in contact with an arbitrary region Γ of the z = 0 surface is equal to t_{m1}. The remaining part of the surface z = 0 is in contact with an external medium of temperature t_m. The heat-transfer coefficient in the region Γ is denoted by α_1, and from the surface z = 0 outside Γ by α, with $\alpha_1 > \alpha$. The surfaces x = 0, $x = x_0$, y = 0, $y = y_0$ are either thermally insulated or are kept at temperature t_m. In dimensionless variables, the boundary-value problem for the determination of the non-steady-state temperature field in the semiinfinite rectangular prism can be written as

$$\frac{\partial^2\theta}{\partial Z^2} + \frac{\partial^2\theta}{\partial X^2} + \frac{\partial^2\theta}{\partial Y^2} = \frac{\partial\theta}{\partial \mathrm{Fo}}, \tag{1}$$

$$\beta_x \left.\frac{\partial\theta}{\partial X}\right|_{x=0} = \gamma_x\theta|_{x=0},\ \ \beta_x \left.\frac{\partial\theta}{\partial X}\right|_{x=\pi} = -\gamma_x\theta|_{x=\pi}, \tag{2}$$

$$\beta_y \left.\frac{\partial\theta}{\partial Y}\right|_{y=0} = \gamma_y\theta|_{y=0},\ \ \beta_y \left.\frac{\partial\theta}{\partial Y}\right|_{y-b} = -\gamma_y\theta|_{y=b},$$

$$|\beta_\zeta|\,|\gamma_\zeta| = 0,\ \ |\beta_\zeta| + |\gamma_\zeta| \neq 0,\ \ \zeta = X,\ Y,$$

$$\frac{\partial\theta}{\partial Z} = \mathrm{Bi}\,\theta + [(\mathrm{Bi}_1 - \mathrm{Bi})\,\theta - \mathrm{Bi}]\,\chi(X,\ Y)\ \text{ for }\ Z = 0, \tag{3}$$

$$\theta|_{t\to\infty} = 0,\ \ \theta|_{\mathrm{Fo}\to 0} = 0, \tag{4}$$

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